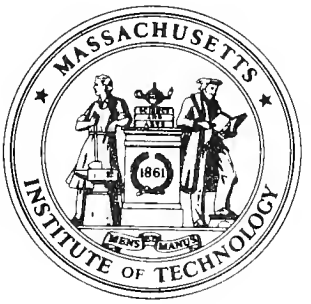

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# KEYNES' DISEQUILIBRIUM ANALYSIS

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Number 88

August 1972

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Keynes' "General Theory" has, with exceptions in the analysis of the multiplier, generally been presented, as Joan Robinson recently put it, within the 'cocoon of equilibrium' (14, p. 3) i.e. as a description of the conditions for macroeconomic (static) equilibrium. In view of the ample reference in the "General Theory" to expectations, lags and dynamic processes this limitation would, at best, appear to be a gross simplification of Keynes' analysis. Hicks stated in his famous review article that "the use of the method of expectations is perhaps the most revolutionary thing about this book." (6, p.240) But in equilibrium, expectations are equal to actual values and hence the adjustment process is lost. Hansen wrote that the fact "that the General Theory is fundamentally a study of the economy in motion, is evident throughout the book." (5, p. 47) But in static equilibrium there is no motion.

Recent studies by Clower (1), Leijonhufvud (11), Hines (8), and Grossman (4) have contributed substantially towards rectifying the bias of static equilibrium interpretation by accentuating the dynamic disequilibrium aspects of Keynes' analysis. (See also 15, pp. 4-11) These studies do however center on specific parts of the entire Keynesian disequilibrium analysis and, in general, suffer from the lack of a formal apparatus into which to cast the dynamic interaction processes. (8, p. 25)

This article is an attempt to give a more comprehensive restatement of Keynes' "General Theory" and to analyse its behaviour under disequilibrium conditions by way of computer simulation. Planned magnitudes will be allowed to deviate from actual magnitudes by introducing them as separate variables; the equilibrium assumption will be replaced, where appropriate, by decision mechanisms through the introduction of lags, feed-back loops and error adjustment processes. The disequilibrium presentation will, of course, comprise the description of the equilibrium situation as a special case.

The specific form of disequilibrisation is undertaken by applying the technique of system dynamics.(3, 13,15) Developed especially for the description of dynamic systems behaviour, it not only provides the tool to take account of lags, feed-back loops and error adjustment processes,

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but, by analysing a system in terms of levels (stock variables), rates (flow variables), conserved flows and information flows, also tends to assure a more comprehensive description of the interaction processes than the usual income-expenditure approach.[1] I have tried to incorporate both Keynes' short-run and his long-run analysis in the model, allowing for their distinction by appropriate time lags, especially as to the variations in capital stock. So as not to overload the presentation I have, however, left out those parts of Keynes' analysis that refer to government activities (other than those of the monetary authorities) and the foreign trade sector. In spite of the fact that it means covering familiar ground, in order to describe the disequilibrium model in its entirety, I have restated all functional relationships between the variables. In a number of cases, rereading Keynes under the aspect of disequilibrium analysis and being freed from the technical restrictions of the usual income-expenditure models, has however led to interpretations different from those in the standard presentations. Only in these cases and where my analysis extends to incorporate otherwise neglected variables, is direct reference made to the "General Theory." Part I analyses the product market, Part II the labour market, and Part III the money market. The entire model is set up and its results demonstrated in Part IV.

## I

The product market divides into the market for consumer goods and for capital equipment. Real demand for consumer goods [1][2] depends firstly on planned (expected) real (net) income (income effect; flow diagram 1). If expected real income falls short of actual real income then, as Clower pointed out (1, esp. p. 120), the latter will act as a budget restraint and actual real income will determine real consumption. Any such difference between expected and actual income will arise concerning the income of entrepreneurs in the case of unforeseen price changes for consumer goods. It can also, and this is the case stressed by Clower, arise in the case of involuntarily unemployed labor, when the thus unemployed, in the expectation of finding employment, are in fact faced with a continuance of their involuntary unemployemnt. But unless the unemployed are expected

1. Some of the more recent work in system dynamics has come under attack from within the economic profession. This critique however applies to the empirical application, not to the technique itself as a theoretical tool of analysis.

2. The numbers in square brackets refer to the numbers of the variables in the flow diagrams and the equation numbers.

to repeatedly make the same planning mistake, irrespective of how often their hopes are disappointed, this factor is not likely to be of great importance.[1]

The real demand for consumer goods will however not reflect instantaneously all changes in income. For in the short run, ("in the case of so-called cyclical fluctuations of employment")...,"habits as distinct from more permanent psychological propensities, are not given time enough to adapt themselves to changed objective circumstances." (9, p. 97) Real consumption is thus dependant on some average of income. Taking account both of these passages, (including the elaborations immediately following), and of Clower's proposition, it seemed appropriate to make present consumption demand, as far as the income effect is concerned, depend on a (weighted) average of past incomes and the expected income for the present period [2]. Since the income of entrepreneurs, rentiers (recipients of contracted capital income), employed labour and unemployed labour all follow different endogenous paths and since the marginal propensity to consume may differ between them it is necessary to distinguish these four groups in the aggregate consumption function.

The real demand for consumer goods depends, secondly, on changes in capital values (real value effect; 9, pp. 92-95; see also 11, pp. 187-205). Assuming expectations to be constant, the interpretation suggested by Leijonhufvud, (11, p. 197) namely related the real non-money asset value generally to the rate of interest would appear justified. If one does however allow for changes in expectations, (9, esp. pp. 146-164), then it becomes necessary to differentiate between bonds and equities. While the price of bonds is related to the rate of interest, (9, p. 96), the price of equities is then subject to the fluctuations of the stock market [3]. I shall return to these points when discussing the market for capital goods.

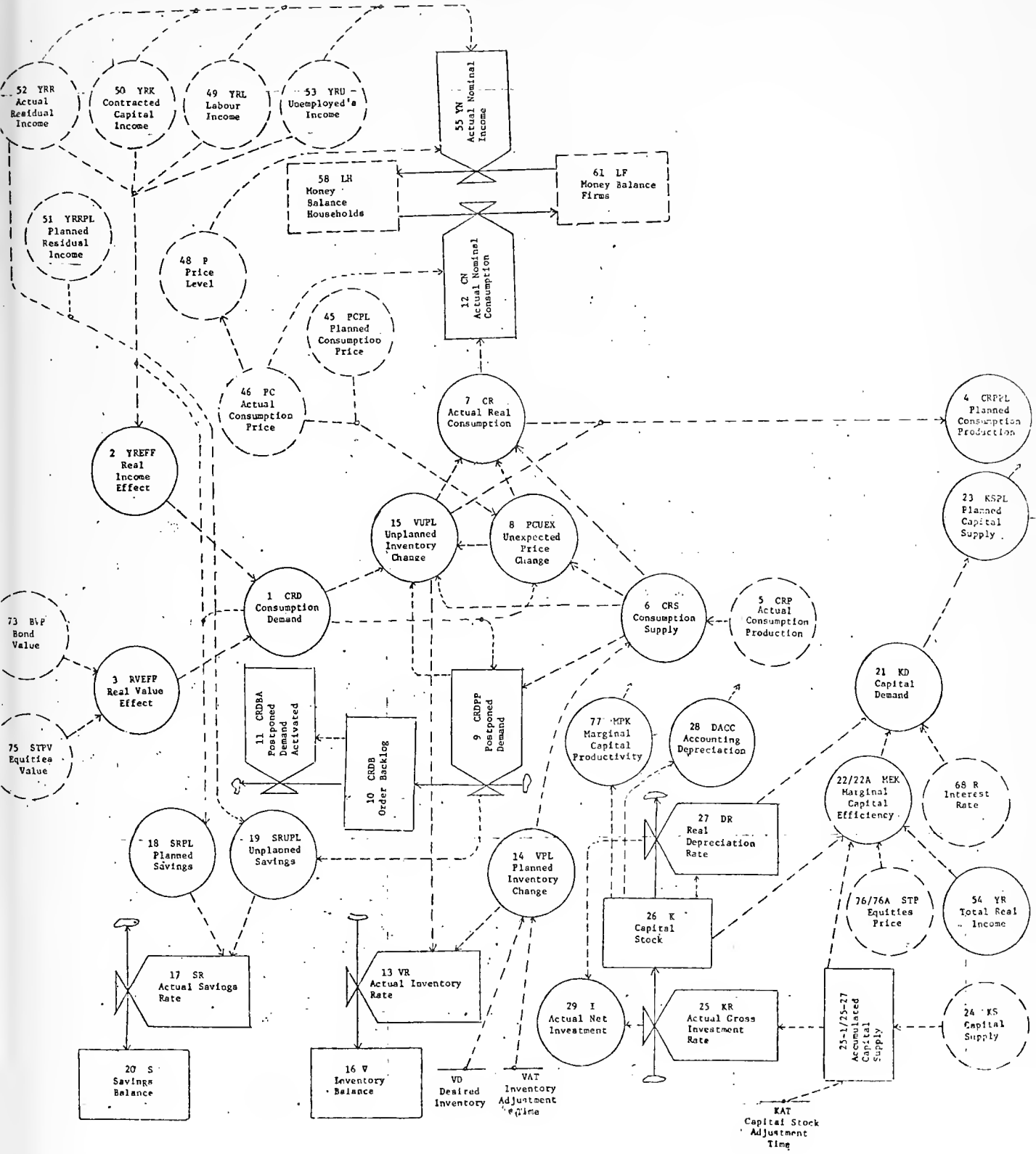
The planned real production of consumer goods [4] depends on short-term expectations of the producers (9, p. 97) and as for these "it is sensible

---

1. Planned and actual real income can furthermore diverge if the expected price level in general differs from the actual price level, and this affects not only entrepreneurs and unemployed labour, but also contracted income, both labour and capital income, for which the expected nominal value is necessarily equal to the actual nominal value. I could, however, find no reference to this in Keynes and hence the expected price level is assumed to be identical with the actual price level (although this is clearly unrealistic in view of unforeseen price changes explicitly referred to by Keynes.)

for producers to base their expectations on the assumption that the most recently realized results will continue, except in so far as there are definite reasons for expecting a change." (9, p. 51) In a stationary state this can be assumed to mean that the planned supply of consumer goods is dependant simply on the actual real sales of the previous period, but in a dynamic state it must, I think, be interpreted to mean that planned real supply equals the sales of the previous period plus an increase (or decrease) proportionate to that of the previous periods. (See also 9, footnote p. 48). The actual real production [5] is determined by the amount of employment and the shape of the cost function in the consumption industry. Together with planned changes in inventory it determines the effective real supply [6].

If there is a deficit or a surplus of real demand for consumer goods over effective real supply within any one period, then unplanned changes in inventory have generally been assumed to make up for the difference, leaving the price within that period unaffected. In his disequilibrium analysis Keynes does however allow also for price changes (which are not due to higher cost as indicated by the supply curve). Thus he states that in the case of an unforeseen increase in investment "the efforts of those newly employed in the capital-goods industries to consume a proportion of their increased income will raise the prices of consumer goods until a temporary equilibrium between demand and supply has been brought about, partly by the high prices causing a postponement of consumption, partly by a redistribution of income in favour of the savings classes as an effect of the increased profits resulting from the higher prices and partly by the higher prices causing a depletion of stocks." (9, pp.123-124) The extent of actual real consumption [7] is thus determined by the price rise [8] itself, the inventory changes due to the price increase and the postponement of demand [9] (changes in entrepreneurial income are reflected in the consumption demand). Keynes gives no indication as to the relative importance of these three factors. I have arbitrarily weighted them by (easily exchangeable) constants. It should be borne in mind for later reference that the price change considered here is not due to an increase in cost and will have to be entered separately when calculating the price level. The postponement of demand in any one period will lead to an order backlog [10] and increase demand in the following period [11]. The unforeseen




Flow Diagram 1

The Product Market

change in inventory is taken account of in the production planning of the next period. Nominal consumption [12] is the product of real consumption times the price of consumer goods.

The actual inventory rate, that is the change of inventory per period [13] is determined by the planned and unplanned inventory change. Planned inventory change [14] equals the difference between desired inventory and actual inventory over inventory adjustment time.(9, pp. 51, 76)[1] Unplanned inventory change [15] is determined by the disequilibrium adjustment process in the market for consumer goods. The inventory balance [16] is the (de)accumulation of the inventory rate over time.

The actual real savings rate [17], savings per period of time, is made up of planned and unplanned real savings. Planned real savings [18] equal the difference between planned real income and consumption demand. Unplanned savings [19] arise firstly from the difference between expected income and actual income (limited to entrepreneurs) and secondly from the difference between planned real consumption and actual real consumption. The savings balance [20] is the accumulation of the savings rate over time.

In the market for capital equipment, real demand for gross investment [21] is determined by the expected marginal efficiency of capital, the rate of interest and the actual rate of depreciation. The marginal efficiency in turn depends on the expected discounted yield and the expected supply price (9, p. 135).[2] Keynes relates a number of variables to the marginal efficiency of capital, (9, pp.141-147) but suggests, that no determinate relationship can be established between any one or any combination of them and the marginal efficiency of capital. Although "enterprise", that is to say the prognosis of the expected yield on the basis of presently available data, will have an important influence on the marginal efficiency of capital (9, pp. 148, 159), other factors, especially the "state of confidence" (9, pp. 148-149) or "speculation"

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1. Of course the inventory change forms part of the investment, but for clarity of exposition it seems useful to deal with it separately.

2. Since the discounted yield is dependant on the rate of interest, the marginal efficiency of capital is itself dependant on the expected rate of interest (9, pp. 137, 143). For simplicity it is assumed that this dependance is accounted for, given the marginal efficiency of capital, by the relationship of the demand for capital equipment and the divergence between the rate of interest and the marginal efficiency of capital. Keynes in the exposition on the marginal efficiency of capital adopts a similarly simplifying procedure (9, p. 149).

(9, p. 158), which elude prognosis, enter to a considerable or even dominant extent into its determination. The changes of the marginal efficiency of capital are reflected, however, in the fluctuations of the stock market (9, pp. 151-152) which may thus serve as an indicator of the former. The fluctuations of the stock market follow no regular course, but show a rather erratic, unpredictable pattern (9, pp. 158-169). Linking investment via the marginal efficiency of capital to these erratic stock market fluctuations, with investment as one of the major determinants for the behaviour of the entire system, would, of course, lead to rather irregular variations of all the dependant variables. In order to demonstrate better the regular relationships between the variables I have hence, following Keynes, excluded these erratic fluctuations in running the model at first by making the marginal efficiency dependant on the ratio of real income over existing capital stock plus capital goods inventory [22]. In subsequent runs I did allow for fluctuations in the marginal efficiency of capital as indicated by the stock market prices, by introducing a noise function for the price of stocks normally distributed around the former value [22A].

Throughout the "General Theory" Keynes would appear to assume the market for capital equipment to be in equilibrium (9, see esp. pp. 135-146). While it would be tempting (and quite easy) to apply the disequilibrium mechanism used for the market for consumer goods also to the market for capital equipment, I shall, in order to follow Keynes' exposition, assume demand equal to planned supply [23] and planned supply equal to effective supply.[1] The effective supply is determined by the cost curve in the capital equipment industry [24]. However, a substantial time lag is introduced between the exertion of demand and the actual investment, (addition to capital stock) [25], thus allowing for the distinction between short-run effects (constant stock of capital equipment) and long-run effects (variable stock of capital equipment). Making the demand for capital equipment

---

1. The disequilibrium assumption in the consumer goods market and the equilibrium assumption in the market for capital equipment lead, of course, to a disproportionate development in both sectors. Such a disproportionate development occurs whatever the disequilibrium adjustment process on both markets, if the two sectors operate under different cost conditions. (See p. 9 ) The equilibrium assumption on the market for capital equipment may thus be viewed as a special case of a fully disequilibrialized system.

equal to the supply, while keeping capital stock constant in the short run, necessarily means that the capital goods produced are temporarily accumulated into the capital goods inventory. In the long run the actual gross (capital equipment) investment rate is accumulated into the stock of capital equipment [26]. The stock of capital equipment is diminished by the real depreciation rate [27]. It increases as stock increases. The depreciation which enters into the accounts, [28], can be larger than the actual depreciation rate (9, pp. 100-101). It is the accounting depreciation which reduces actual residual gross income to actual residual net income, (9, p. 104) which in turn determines entrepreneurial consumption. As a drain on aggregate demand it may be of considerable importance "during a period which immediately succeeds a lively burst of investment in long-lived capital" (9, p. 100). Real net capital investment [29] equals the effective real supply of capital goods minus the real capital depreciation rate.

## II

As to the labour market, Keynes carries out most of his analysis in terms of a one sector model. He does however deal explicitly with different cost functions in the consumer goods and the capital goods industry (9, pp. 43, 116, 282, esp. 286-289) and this differentiation can be important in the determination of total employment (9, p. 286), the distribution of income (9, p. 287), and the relative prices (9, p. 288). The differentiation between the two sectors becomes especially important when the consumer goods market is in disequilibrium while the market for capital goods is in equilibrium. The same would hold if both markets were in disequilibrium, but with different market adjustment mechanisms operating on each.

The demand for labour depends on effective product demand and the marginal product of labour (9, pp. 25, 29, 98, 280-286).[1] Effective demand

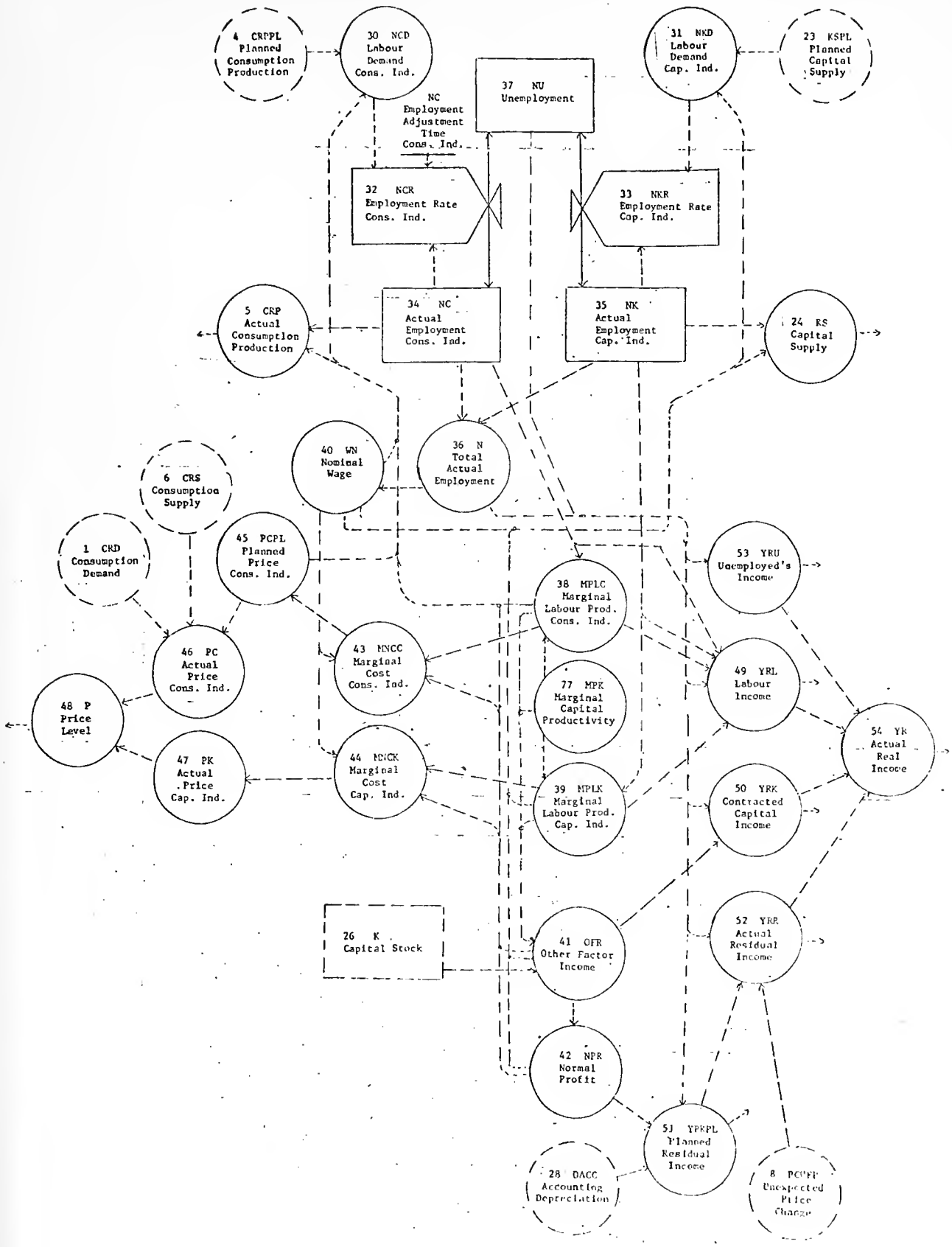
1. In his employment function Keynes relates the demand for labour only to the effective product demand. The dependance on the marginal product of labour follows by way of necessity from the fact that the employment function is the inverse of the aggregate supply function (9, p. 280), representing marginal cost which, in turn, is dependant on the marginal productivity of labour.

in the consumption industry [30] is determined by the planned production of consumer goods, effective demand in the capital industry [31] by planned gross (including reinvestment) production of capital equipment (flow diagram 2). Actual employment will however not adjust to the desired employment instantaneously. Rather the employment rate, that is the actual change in employment per unit period, will be high at first and decrease, ceteris paribus, gradually as the difference between actual employment and desired employment decreases (9, pp.47-48). An appropriate mechanism, following Keynes' description of the adjustment process, has been built into the model. In order to be consistent with other parts of the model, it is effective however only in the consumer goods industry [32], whereas the equilibrium assumption in the capital goods industry implies an adjustment time so short that it can be neglected [33].

Employment in the consumer goods [34] and in the capital goods industry [35] together add up to total employment [36]. The difference between the total available labour force and the actual total employment gives the number of unemployed [37], receiving unemployment benefit. The unemployment thus defined is dependant on the existence of persons able to work. It is not, but by chance, identical with involuntary unemployment, which is dependant on the willingness to work.[1]

The actual level of employment in the consumer and the capital goods industry determines the respective marginal productivity of labour [38, 39] in the short run, which is assumed decreasing in both sectors as employment increases. Keynes, being concerned predominantly with short-run analysis, assumes the stock of capital equipment constant throughout most of his analysis. He does however operate with variations thereof in discussing the supply curve (9, p. 136) and also in his long-run analysis (9, pp. 212, 217-224). In the long run, the marginal product of labour will therefore also depend on capital stock, increasing as capital stock increases, but at a

1. The extent of involuntary unemployment is of no importance to the performance of the system so that the issue is not further pursued here.




Flow Diagram 2

The Labour Market

decreasing rate.

The nominal wage [40] is the result of bargaining between entrepreneurs and trade unions (9, p. 11). According to Keynes it is constant for low levels of employment, but will rise as the full employment situation is approached.[1] The nominal cost of other factors [41] employed and normal profit (9, pp. 68, 72, 77) [42] is given by a constant factor to the initial nominal wage rate (9, p. 55).[2] Changes in the nominal wage rate leave the nominal cost of other factors unaffected (9, pp. 262, 290). This factor can, however, only be assumed constant in the short run when the stock of capital (hence also the stock of industrial bonds and equities) is fixed. In the long run it will show a tendency to rise as the stock of capital increases and a tendency to fall as the marginal productivity of capital declines. Although Keynes makes no specific reference to the long-run variations of this factor, these conclusions follow by way of necessity from the fact that capital income, given the income per capital unit, must increase as capital stock rises, but will fall as the decreasing scarcity of capital will lower the income per unit (9, pp. 375-377).[3] The nominal wage rate and the nominal cost of other factors including normal profit are used in deriving nominal marginal cost [43, 44]. Nominal marginal cost is equal, assuming of course perfect competition, to the planned price in the two sectors (9, pp.114, 173). In the consumer goods industry expected price [45] is increased or reduced by the unexpected price change, as results from the consumer goods market, to yield the actual price [46].[4] The planned price in the capital goods

---

1. Nominal wage will start rising from the point at which the disutility of labor in terms of money becomes equal to the nominal wage rate (9, pp. 10, 15-17).

2. Since that part of capital income paid to bondholders is precontracted, the residual income left as dividend to owners of equities will show an amplitude of variation that is relatively greater than that of employment itself.

3. In the very long run, "within one or two generations" Keynes did in fact expect the scarcity of capital to have sunk to a level at which capital income as such becomes zero (9, p. 377).

4. Keynes is of course caught in the general dilemma of the perfect competition model, because if all market participants are price takers, then who is left to actually change the price?

industry is again, in view of the equilibrium assumption, equal to the actual price [47]. The actual prices in the consumer and capital goods industries, weighted by the actual turnover, determine the price level [48].[1]

The product of total employment times the marginal product of labour gives contracted real labour income [49]. Employment times the real income factor for other factors of production excluding normal profit gives contracted real capital income [50]. Employment times real normal profit minus the accounting depreciation (9, pp. 98-99) determines planned residual net income [51]. The deduction of the unexpected price changes in the consumer goods market (9, pp. 121, 127) yields real net residual income [52]. The amount of transfer income to the unemployed [53] is assumed to depend on the level of unemployment, as defined above. Real actual contracted (labour- and capital-) and residual income after redistribution plus real transfer income to the unemployed adds up to total actual real income [54], which, of course, is the primary determinant of real consumption demand. Nominal income [55] is derived by multiplication with the price level.

Having thus closed the loop back to the demand for consumer goods, it becomes evident that the disequilibrium analysis entails the mutual dependance of product market disequilibrium on labor market disequilibrium and vice versa. It thus comprises both Patinkin's exposition (12, pp. 316-327) whereby involuntary unemployment on the labor market is dependant on deficient demand on the product market and Clower's presentation, adopted by Leijonhufvud, whereby deficient demand in the product market is the direct result of involuntary unemployment in the labor market. But while Keynes' analysis does in fact incorporate both aspects, I agree with Grossman (5, esp. p. 28) that there is no evidence in the "General Theory" that Keynes should have viewed this interdependance as based on the choice-theoretical conception suggested by Clower and Leijonhufvud.

---

1. Nowhere in the "General Theory" is the price level actually defined (see also 8, p. 26). But with prices being determined on the one hand by cost-push in the production sphere and by demand-pull in the market, it seems logical that both considerations should enter into the determination of the price level.

## III

On the money market the desired money balances by households are composed of the desired money balance for transactions (including precautionary purposes) [56] and the desired money balance for speculation [57] (flow diagram 3). The desired money balance for transactions relates to total actual nominal income, (9, pp.170, 195-196), by the velocity of money which is assumed a parametric constant (9, p. 201).[1] The desired money balance for speculations relates to the rate of interest (9, pp. 168, 197, 204).

The actual money balance held by the households [58], in the absence of operations in the money market, is the result of money inflows from income and money outflows for consumption. The difference in the desired money balance and the actual money balance determines the desired change in money balances [59], thus leading to an equivalent supply of money (assuming aggregate savings positive). The transactions on the product market give rise to continuous changes in the actual money balance and will thus lead to a continuous supply of money by households (assuming positive savings).

The desired money balance by firms [60] depends only on total nominal income (in proportion to the velocity of money) since it is assumed that the firms do not engage in speculation. Their actual money balance [61] would, again in the absence of operations on the money market, be determined by inflows from the sale of consumer goods and outflows for paying the factors of production. The difference between their desired and their actual money balance, determines the desired change in money balances by firms [62], thus leading to an equivalent demand for money (assuming again positive savings). Any demand for money by firms that is matched by an equivalent supply of money by households or vice versa can be absorbed by cashflows between the two sectors [63, 63A, 63B].

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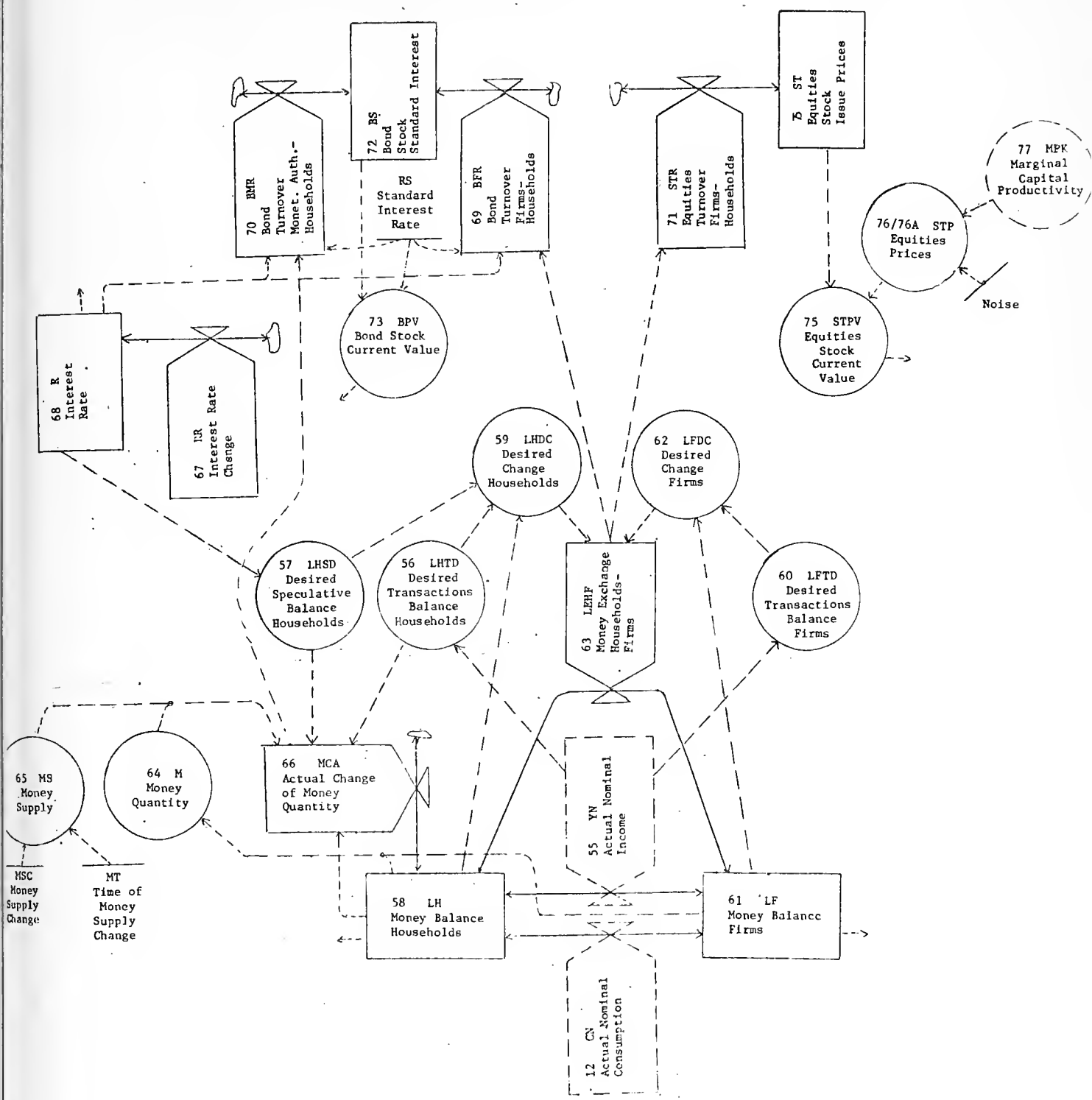
1. Linking the money balance for transactions to actual money balances is unsatisfactory in the sense that it does not include money demand for expected rises in income, especially planned investment; but Keynes later (10, p. 667) explicitly admitted to having left out this aspect in the "General Theory." For a more extensive treatment see (2). It is the dependance of the transactions balance on actual rather than expected income that constitutes the fundamental difference between the liquidity preference and the loanable fund approach (Compare 8, pp. 43-48).




Flow Diagram 3

The Money Market

The total amount of money [64] is of course given by the actual money balances of households and firms. Any excess demand or supply over the quantity of money available will be equilibrated by changes in the rate of interest, [67, 68], increases in the rate of interest reducing (speculative) demand and vice versa.

The cash transfer from households to firms will imply an equivalent transfer of bonds [69] or equities [71] in the opposite direction. So as to make the flow of bonds from different periods (with changing interest rates) comparable, the nominal bond flow has been reduced to a standard flow of bonds at a standard rate of interest. The accumulated standardized stock of bonds [72] valued in terms of the prevailing interest rate gives the stock of bonds at current prices[73]. The accumulated stock of equities [74], on the other hand, is assumed, in the simple model, (marginal efficiency of capital dependant on the ratio of real income and capital stock), to be priced equal to the marginal productivity of capital [75, 76], in the extended model by the same noise function that determines the marginal efficiency of capital.

If the excess demand for money is matched by an equal supply of bonds the interest rate can alternatively be said to depend on the excess supply of bonds. This relation may help to clarify the difference between the loanable fund and the liquidity preference approach to changes in the rate of interest. The former states that any expected rise in the firms' expenditure, such as an increase in investment, will cause firms first to issue bonds in order to obtain the liquid funds necessary to perform the planned transactions, thus having an immediate influence on the rate of interest. The latter allegedly postulates that the interest rate will be affected only after investment has taken place, namely after the increased investment has led to a rise in income and hence in the transactions demand for money (8, pp. 40-47). As Hines points out, the difference would disappear if the transactions demand for money depended on planned rather than actual income. But even with transactions demand dependant on actual income, the loanable fund and the liquidity preference approach tend to lead towards the same result if the increase in the firms' expenditure is reflected in the expected rate of interest. It is the rise in the speculative demand for money that will then drive up the rate of interest before (or during) the increased investment is taking place.

If the monetary authorities want to increase the quantity of money they do so by offering money [65] in exchange for bonds [70]. This will lead to an excess demand for money and an excess supply of bonds. The ensuing fall in the rate of interest will cause households, adjusting their speculative balances to the new situation, to increase their money balances and to decrease their bondholdings. Whether the increased supply of bonds by the monetary authorities will in fact lead to an increase in the quantity of money [66, 66A] depends on the willingness of households to sell bonds or to increase their speculative balances at the decreased interest rate.

IV

The exact formulation of the model is set up for immediate computer simulation. The subscripts J, K, and L refer to past, present and future points in time; JK and KL to rates of change during the intermittent intervals. Constants have no time subscript. The numbers of the equations correspond to the numbers in the flow diagrams. Appendix A gives a list of all symbols used. Appendix B gives a list of all constants, adjustment times, initial values, numerical and purely technical functions used in actually running the model.

## I. Product Market

### Consumption

Eqn. No.

1 CRD.K=YREFF.K+RVEFF.K+CRDBA.JK
2 YREFF.K=C1+C2*YRRAV.K+C3*YRKAV.K+C4*YRLAV.K+C5*YRUAV.K
3 RVEFF.K=C6*(BPV.K)+C7*(STPV.K)
4 CRPPL.K=CR.K(1+CRGR.K)-VPL.U
5 CRP.K=NC.K*C8*(MPLC.K*WN.K+(OPR.K*NPR.K)*(WNO/WN.K))/PCPL.K
6 CRS.K=CRP.K-VPL.K
7 CR.K=CRS.K+PCUEX.K-VUPL.K
8 PCUEX.K=(CRD.K-CRS.K)*(PC.K-PCPL.K)
9 CRDPP.KL=C9*(CRD.K-CRS.K)/CRD.K
10 CRDB.K=CRDB.J+(DT)*(CRDPP.JK-CRDBA.JK)
11 CRDBA.KL=CRDB.K
12 CN.KL=CR.K*PC.K

Inventory

```
13  VR.KL=VPL.K+VUPL,K
14  VPL.K=(VD-V.K)/VAT
15  VUPL.K=CRS.K-CRDPP.JK-CRD.K-PCUEX.K
16  V.K=V.J+(DT)*(VR.JK)
```

Savings

```
17  SR.KL=SRPL.K+SRUPL.K
18  SRPL.K=YR.K+YRRPL.K-YRR.K-CRD.K
19  SRUPL.K=YRR.K-YRRPL.K+CRDPP.JK
20  S.K=S.J+(DT)*(SR.JK)
```

Capital Equipment

```
21  KD.K=CLIP(C10*(MEK.K-R.K),-DR.JK+1,C9*(MEK.K-R.K),-DR.JK)+DR.JK
22  MEK.K=C11*YR.K/(K.K+KSACC.K)
23  KSPL.K=KD.K*(1+KDGR.K*NKAT)
24  KS.K=NK.K*C8*(MPLK.K+(OFR.K+NPR.K)*(WNO/WN.K))
25  KR.KL=LAG(KS.K-DR.JK,KAT,KSACC.K)+DR.JK
26  K.K=K.J+(DT)*(KR.JK-DR.JK)
27  DR.KL=C13*K.K
28  DACC.K=C14*K.K
29  I.K=KS.K-DR.JK
```

II. Labour Market

Employment

```
30  NCD.K=(CRPPL.K)*PCPL.K/((WN.K*MPLC.K)+(OFR.K+NPR.K)*(WNO/WN.K))*C8
31  NKD.K=KSPL.K/(MPLK.K+(OFR.K+NPR.K)*(WNO/WN.K))*C8
32  NCR.KL=CLIP((NCD.K-NC.K)/NCAT,0,NU.K,0)[1]
33  NKR.KL=(NKD.K-NK.K)/NKAT
34  NC.K=NC.J+(DT)*(NCR.JK)
35  NK.K=NK.J+(DT)*(NKR.JK)
36  N.K=NC.K+NK.K
37  NU.K=NU.J+(DT)*(-NCR.JK-NKR.JK)
```

Production

```
38  MPLC.K=TABLE(FIG1,NC.K,0,1000,200)*1/MPK.K[2]
```

---

1. The arguments in the clip function state that for a function X = CLIP(P,Q,R,S) that

   X = P if R $\geq$ S and

   X = Q if R $<$ S

2. The arguments in the numerical table function refer to the name of the function, the independant variable, the lowest value of the independant variable, its highest value, and the interval at which values for the dependant variables are given. The numerical functions are specified in Appendix B.

```
39    MPLK.K=TABLE(FIG2,NK.K,0,1000,200)*1/MPK.K
40    WN.K=TABLE(FIG2,N.K,0,1000,200)
41    OFR.K=C15*K.K*MPK.K(MPLC.K*NC.K+MPLK.K*NK.K)/(2*(NC.K+NK.K))
42    NPR.K=C16*OFR.K
43    MNCC.K=1/(MPLC.K*(WNO/WN.K)+OFR.K+NPR.K)
44    MNCK.K=1/(MPLK.K*(WNO/WN.K)+OFR.K+NPR.K)
45    PCPL.K=MNCC.K
46    PC.K=PCPL.K+C17*(CRD.K-CRS.K)/CRD.K
47    PK.K=MNCK.K
48    P.K=(PC.K*CR.K+PK.K*KR.JK)/(CR.K+KR.JK)
```

Income

```
49    YRL.K=(NC.K*C8*MPLC.K+NK.K*C8*MPLK.K)*(1-NU.K*C18)
50    YRK.K=(N.K*C8*OFR.K*(WNO/WN.K))*(1-NU.K*C18)
51    YRRPL.K=(N.K*C8*NPR.K-DACC.K)*(1-NU.K*C18)
52    YRR.K=(N.K*C8*NPR.K*(WNO/WN.K)-DACC.K+PCUEX.K)*(1-NU.K*C18)
53    YRU.K=((NC.K*C8*MPLC.K+N.K*C8*MPLK.K)+(N.K*C8*OFR.K)
      X  +(N.K*C8*NPR.K-DACC.K+PCUEX.K))*(NU.K*C18)
54    YR.K=YRR.K+YRK.K+YRL.K+YRU.K
55    YN.KL=YR.K*P.K
```

III. Money Market

Money Demand

```
56    LHTD.K=YN.JK*1/MV
57    LHSD.K=C20*(R.K)EXP(-3)
58    LH.K=LH.J+(DT)*(YN.JK-CN.JK-LEHF.JK+MCA.JK)
59    LHDC.K=LHTD.K+LHSD.K-LH.K
60    LFTD.K=YN.JK*1/MV
61    LF.K=LF.J+(DT)*(CN.JK-YN.JK+LEHF.JK)
62    LFDC.K=LFTD.K-LF.K
63    LEHF.KL=CLIP(LAUX1.K,0,0,LHDC.K*LFDC.K)[1]
63A   LAUX1.K=CLIP(+LAUX2.K,-LAUX2.K,LHDC.K-LFDC.K,0)
63B   LAUX2.K=CLIP(-MAX(LHDC.K,LFDC.K),MIN(LHDC.K,LFDC.K),0,LHDC.K+LFDC.K)
```

Money Supply

```
64    M.K=LH.K+LF.K
65    MS.K=MN+STEP(MSC,MT)[2]
66    MCA.KL=MIN(MS.K-M.K,CLIP(LAUX3.K,0,LAUX3.K,0))
66A   LAUX3.K=LHSD.K-(LH.K-LHTD.K)
```

Interest

```
67    RR.KL=C23*(LHTD.K+LHSD.K+LFTD.K-MS.K)
68    R.K=R.J+(DT)*(RR.JK)
```

---

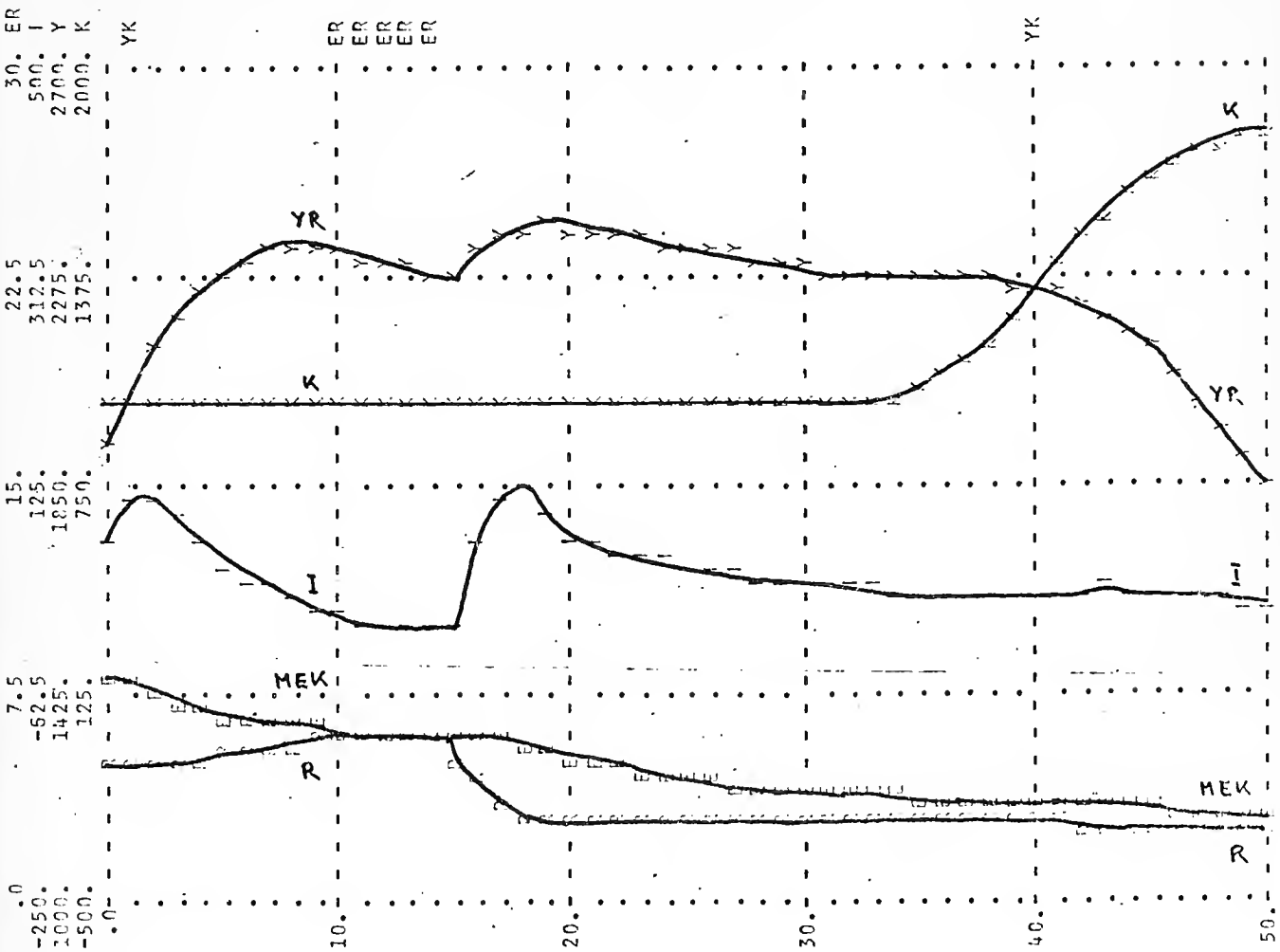
1. The functions for LEHF.K simply assure that any demand for money by firms which is matched by a supply of money by households or vice versa leads to an equivalent exchange of money between the two sectors. Since, depending on the sign of the changes desired by firms and households and on their relative magnitude, this involves eight possibilities, it leads to a somewhat cumbersome construction.

2. The step function means that the supply of money is constant at MN until time MT, and is increased by MSC thereafter.

```
69    BFR.KL=CLIP(LAUX1.K,0,0,LHDC.K*LFDC.K)*C21*R.K/RS
70    BMR.KL=-MCA.JK*R.K/RS
71    STR.KL=CLIP(LAUX1.K,0,0,LHDC.K*LFDC.K)*(1-C21)
72    BS.K=BS.J+(DT)*(BFR.JK+BMR.JK)
73    BPV.K=BS.K*RS/(R.K)
74    ST.K=ST.J+(DT)*(STR.JK)
75    STPV.K=ST.K*STP.K
76    STP.K=MPK.K
77    MPK.K=TABLE(FIG3,K.K,0,5000,1000)
```

The main variables, the interest rate, the marginal efficiency of capital, net investment, capital stock and real income have been plotted for 50 time periods.

RUN 1



The system starts in disequilibrium, with a considerable excess demand for consumer goods over supply. The adjustment process in the consumption industry, together with a rise in net investment leads first to an increase in real income. Due to the lagged adjustment both of inventory and employment (in the consumption industry) real income initially overshoots its equilibrium value which it approaches at period 14. The equilibrium process is marked in the investment sector by the trend towards constant net investment, and a decrease in the difference between the marginal efficiency of capital and the interest rate. At period 15 the monetary authorities are assumed to increase the supply of money. The ensuing fall in the rate of interest leads to a sharp increase in investment and hence real income. The increase is decelerated by the fall in the marginal efficiency of capital leading to a new equilibrium state. From period 30 onwards the investment undertaken during the period of the first increase in investment begins to actually increase the capital stock. The resulting fall in the demand for labour causes real income to decline. The decline is not checked by a decrease in the rate of interest, since interest has reached that level at which the demand for money becomes almost infinitely elastic. Only as the increase in capital stock declines does the decrease in income slow down, the following depreciation of capital stock with the resulting increase in the marginal efficiency of capital being germinative of a renewed upswing.

The general pattern shown here would change fundamentally if adjustment times smaller than the unit time period were introduced. The dynamic behaviour would then show oscillations of the lagged variables. If all adjustment times are equal to one, (no delays), and if furthermore all initial values equal the (initially) desired or expected values then the system would be in equilibrium at the outset and remain so.

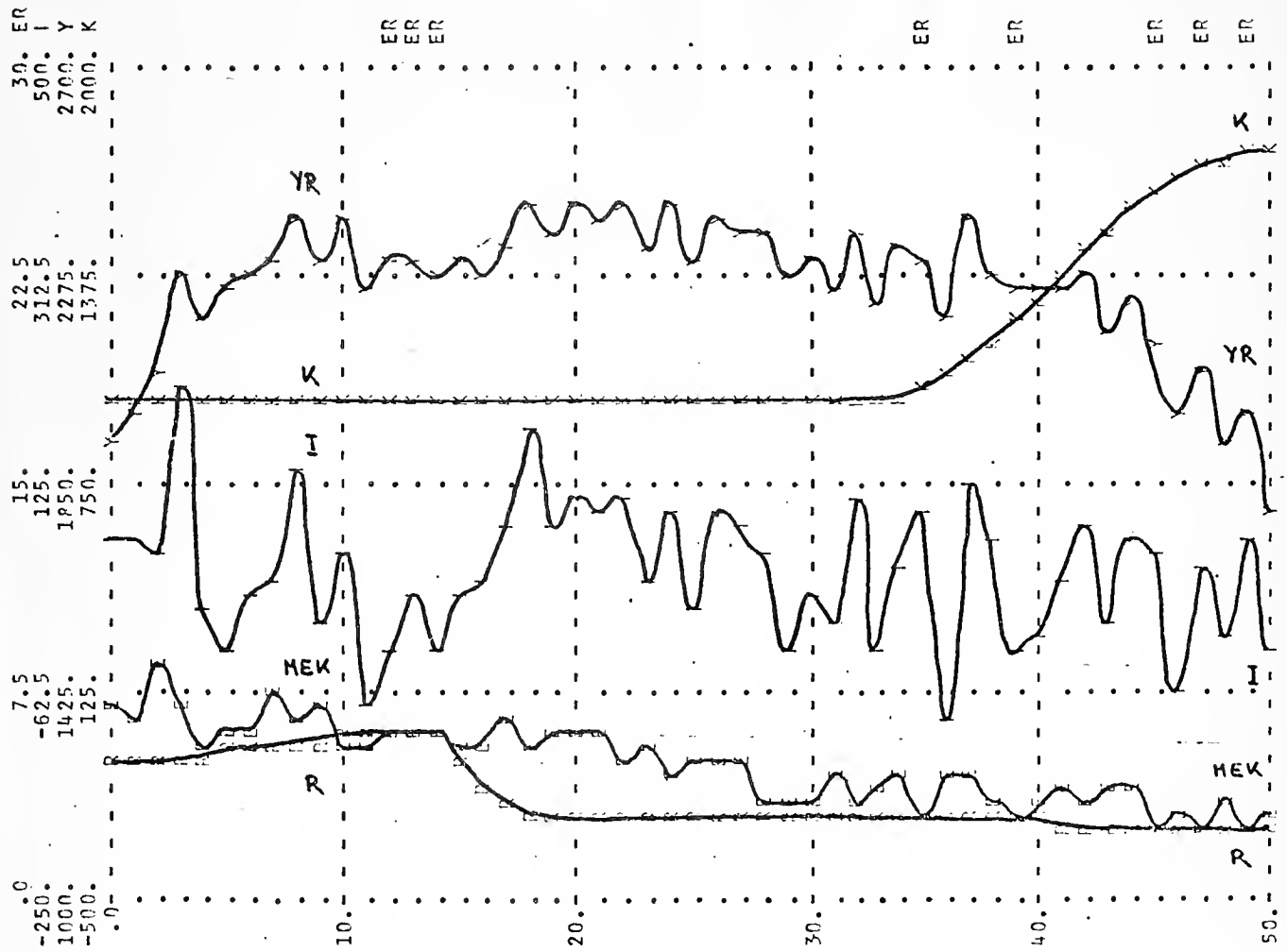
Fluctuating stock market prices as indicating fluctuations in the marginal efficiency of capital have been introduced by rewriting equations 22 and 76.

```
22A . MEK.K=C12*NOISE()+C11*YR.K/(K.K+KSACC.K)
76A   STP.K=C22+NOISE()
```

With the difference between the marginal efficiency of capital and the

interest rate fluctuating irregularly, investment and real income also reflect these fluctuations, while, however, following the general pattern of the previous run.

RUN 2



| Eqn. No. | | |
|---|---|---|
| 69 | BFR | Bond turnover between firms and households valued at standard rate of interest. |
| 70 | BMR | Bond turnover between monetary authorities and household valued at standard rate of interest. |
| 73 | BPV | Stock of bonds valued at present rate of interest. |
| 72 | BS | Stock of bonds valued at standard rate of interest. |
| | C1,...,C23 | Constants. |
| 12 | CN | Nominal actual consumption. |
| 7 | CR | Real actual consumption. |
| 4-1 | CRAV | Average real actual consumption. |
| 4-2 | CRGR | Growth rate of real actual consumption. |
| 1 | CRD | Real consumption demand. |
| 10 | CRDB | Accumulated backlog from postponed real consumption demand. |
| 11 | CRDBA | Activated backlog of postponed real consumption demand. |
| 9 | CRDPP | Postponed real demand for consumption per period. |
| 5 | CRP | Actual real production in consumption industry. |
| 4 | CRPPL | Planned real production in consumption industry. |
| 6 | CRS | Effective real consumption supply. |
| 27 | DR | Actual real depreciation rate. |
| 28 | DACC | Accounting depreciation. |
| - | DT | Unit time period. |
| 29 | I | Actual real net capital investment rate. |
| 26 | K | Real capital (equipment) stock. |
| - | KAT | Capital stock adjustment time (lag between capital investment and increase of capital stock). |
| 21 | KD | Real capital demand for gross investment. |
| 23-1 | KAAV | Average real capital demand. |
| 23-2 | KDGR | Real capital demand growth rate. |
| 25 | KR | Actual real gross capital investment rate. |
| 24 | KS | Effective real capital supply. |
| 25-25 | KSACC | Accumulated capital supply (capital goods inventory). |
| 23 | KSPL | Planned real capital supply. |
| 63 | LEHF | Liquidity exchange rate between households and firms. |
| 61 | LF | Actual money balance held by firms. |
| 62 | LFDC | Desired change in money balance held by firms. |

APPENDIX A - 2

| | | |
|---|---|---|
| 60 | LFTD | Desired transactions (money) balance held by firms. |
| 58 | LH | Actual money balance held by households. |
| 59 | LHDC | Desired change in money balance held by households. |
| 57 | LHSD | Desired speculative (money) balance held by households. |
| 56 | LHTD | Desired transactions (money) balance held by households. |
| 64 | M | Quantity of money. |
| - | MN | Initial quantity of money. |
| 66 | MCA | Actual change in the quantity of money. |
| 65 | MS | Money supply. |
| - | MSC | Change in money supply. |
| 22 | MEK | Marginal efficiency of capital. |
| 77 | MPK | Marginal productivity of capital. |
| 38 | MPLC | Marginal productivity of labour in consumption industry. |
| 39 | MPLK | Marginal productivity of labour in capital industry. |
| 43 | MNCC | Marginal nominal cost in consumption industry. |
| 44 | MNCK | Marginal nominal cost in capital industry. |
| - | MT | Period at which money supply is changed. |
| - | MV | Money velocity. |
| 36 | N | Total actual employment. |
| 34 | NC | Actual employment in consumption industry. |
| - | NCAT | Employment adjustment time in consumption industry. |
| 30 | NCD | Labour demand from consumption industry. |
| 32 | NCR | Employment rate (change of employment) in consumption industry. |
| 35 | NK | Actual employment in capital industry. |
| - | NKAT | Employment adjustment time in capital industry. |
| 31 | NKD | Labour demand from capital industry. |
| 33 | NKR | Employment rate (change of employment) in capital industry. |
| 42 | NPR | Normal profit rate. |
| 37 | NU | Unemployment. |
| 41 | OFR | Income rate paid to contracted factors of production other than labour (contracted capital income). |
| 48 | P | Price level. |
| 46 | PC | Actual price in consumption industry. |
| 45 | PCPL | Planned (expected) price in consumption industry. |

APPENDIX A - 3

| | | |
|---|---|---|
| 8 | PCUEX | Unexpected price change in consumption industry. |
| 47 | PK | Actual price in capital industry. |
| 68 | R | Actual rate of interest. |
| 67 | RR | Change in the actual rate of interest. |
| - | RS | Standard rate of interest. |
| 3 | RVEFF | Real value effect on consumption. |
| 20 | S | Real savings balance. |
| 17 | SR | Actual real savings rate. |
| 18 | SRPL | Planned real savings. |
| 19 | SPUPL | Unplanned real savings. |
| 74 | ST | Stock of equities held by households at issue prices. |
| 76 | STP | Price of equities. |
| 75 | STPV | Stock of equities valued at present prices. |
| 71 | STR | Equities turnover at issue prices. |
| 16 | V | Inventory balance. |
| - | VAT | Inventory adjustment time. |
| - | VD | Desired inventory. |
| 13 | VR | Actual inventory rate (actual change in inventory per period). |
| 14 | VPL | Planned inventory change. |
| 15 | VUPL | Unplanned inventory change. |
| 40 | WN | Nominal wage. |
| - | WNO | Initial nominal wage. |
| 55 | YN | Total actual nominal income. |
| 54 | YR | Total actual real income. |
| 2 | YREFF | Real income effect on consumption |
| 50 | YRK | Actual real contracted capital income after redistribution. |
| 50-1 | YRKAV | Average real contracted capital income. |
| 49 | YRL | Actual real labour income after redistribution. |
| 49-1 | YRLAV | Average real labour income. |
| 52 | YRR | Actual real residual income. |
| 52-1 | YRRAP | Average actual past real residual income. |
| 52-3 | YRRAV | Average presently planned and actual past real residual income. |
| 51 | YRRPL | Planned real residual income. |
| 53 | YRU | Actual real transfer income to unemployed. |
| 53-1 | YRUAV | Average real transfer income to unemployed. |

# APPENDIX B

Constants:

C1 = 500
C2 = 0.5
C3 = 0.5
C4 = 0.7
C5 = 1.0
C6 = 0.01
C7 = 0.01
C8 = 10
C9 = 100
C10 = 40
C11 = 5
C12 = 2
C13 = 0.1
C14 = 0.11
C15 = 0.00025
C16 = 1
C17 = 1.5
C18 = 0.001
C19 = 0.2
C20 = 10,000
C21 = 0.5
C22 = 1
C23 = 0.003
MV = 7.4
MSC = 300
MT = 15
RS = 5
DT = 1

Adjustment times:

VAT = 4
KAT = 40
NCAT = 4
NKAT = 1

Initial values:

CRAV = CR
CRDB = 0
VD = 100
V = 100
S = 100
KDAV = KD
K = 1000
NC = 140
NK = 15
NU = 100
WNO = 1
LH = 300
LF = 300
MN = 600
BS = 200
ST = 200
R = 5

Numerical functions:

Fig. 1 = 0.8, 0.8, 0.7, 0.6, 0.5, 0.5
Fig. 2 = 1, 1, 1, 2, 4, 10
Fig. 3 = 0.9, 0.85, 0.8, 0.7, 0.5, 0.1

Technical equations:

```
4-1     CRAV.K=CRAV.J+(DT)*(CR.J-CRAV.J)
4-2     CRGR.K=(CR.K-CRAV.K)/CR.K

23-1    KDAV.K=KDAV.J+(DT)*(KD.J-KDAV.J)/(NKAT)
23-2    KDGR.K=(KD.K-KDAV.K)/(KD.K*NKAT)

        Macro Lag (input, delayt, cont)

25-1    $R1.KL=KELAY3P(INPUT.JK,SIND.K,$L1.K)
25-2    $R2.KL=DELAY3P($R1.JK,$IND.K,$L2.K)
25-3    $R3.KL=DELAY3P($R2.JK,$IND.K,$L3.K)
25-4    $R4.KL=DELAY3P($R3.JK,$IND.K,$L4.K)
25-5    $R5.KL=DELAY3P($R4.JK,$IND.K,$L5.K)
25-6    $R6.KL=DELAY3P($R5.JK,$IND.K,$L6.K)
25-7    $R7.KL=DELAY3P($R6.JK,$IND.K,$L7.K)
25-8    $R8.KL=DELAY3P($R7.JK,$IND.K,$L8.K)
25-9    $R9.KL=DELAY3P($R8.JK,$IND.K,$L9.K)
25-10   $R10.KL=DELAY3P($R9.JK,$IND.K,$L10.K)
25-11   $R11.KL=DELAY3P($R10.JK,$IND.K,$L11.K)
25-12   $R12.KL=DELAY3P($R11.JK,$IND.K,$L12.K)
```

APPENDIX B - 2

```
25-13     $R1=0
25-14     $R2=0
25-15     $R3=0
25-16     $R4=0
25-17     $R5=0
25-18     $R6=0
25-19     $R7=0
25-20     $R8=0
25-21     $R9=0
25-22     $R10=0
25-23     $R11=0
25-24     $R12=0
25-25     CONT.K=$L1.K+$L2.K+$L3.K+$L4.K+$L5.K+$L6.K+$L7.K+$L8.K+$L9.K+$L10.K+
          X  $L11.K+$L12.K
25-26     LAG.K=$L12.K/$IND.K
25-27     $IND.K=DELAYT/12.0

          Mend

49-1      YRLAV.K=YRLAV.J+(DT)*(YRL.J-YRLAV.J)
49-2      YRLAV=YRL
50-1      YRKAV.K=YRKAV.J+(DT)*(YRK.J-YRKAV.J)
50-2      YRKAV=YRK
52-1      YRRAP.K=YRRAP.J+(DT)*(YRR.J-YRRAP.J)
52-2      YRRAP=100
52-3      YRRAV.K=C20*(YRRPL.K)+(1-C20)*(YRRAP.K)
53-1      YRUAV.K=YRUAV.J+(DT)*(YRU.J-YRUAV.J)
53-2      YRUAV=500
```

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